

Solve the following problems: Calculators are not allowed.

1. a) Solve the inequality $\frac{x^2 - 3x + 2}{3 - x} \leq 0$
- b) Find the following limits (if they exist)
- $$\lim_{\theta \rightarrow 0} \frac{\sqrt{1 + \sin \theta} - 1}{\theta}, \quad \lim_{x \rightarrow -\infty} \frac{|x|}{\sqrt{4x^2 + 3}} \quad (6 \text{ marks})$$
2. Let $f(x) = \begin{cases} x^2 + 2x + 1 & \text{if } x \leq 0 \\ 1 + \sin x & \text{if } x > 0 \end{cases}$
- a) show that f is continuous at $x = 0$
- b) Is f differentiable at $x = 0$? (6 marks)
3. a) Find the equation of the normal line to the graph of $y^3 + xy^2 + x^2 + 3 = 0$ at the point $(1, -2)$
- b) Let $y = \frac{x}{x^2 + 2}$. Use differentials to approximate the change in y when x changes from 1 to 1.01. (6 marks)
4. Let $f(x) = \tan x + 4x - 3$.
- a) Prove that $f(x) = 0$ has a solution between 0 and $\frac{\pi}{4}$.
- b) Use Rolle's theorem to prove that $f(x) = 0$ has only one solution in $(0, \frac{\pi}{4})$ (6 marks)
5. A rectangular garden of area 75 ft² is bounded on three sides by a wall costing \$8 per ft and on the fourth side by a fence costing \$4 per ft. What are the dimensions of the garden for minimum cost. (6 marks)
6. a) Find $\int_0^{\pi/2} \cos x \sin^5 x \, dx$
- b) Find $\int \frac{(4x-2)}{\sqrt{x^2-1}} \, dx$ (6 marks)
7. a) Find the average value of $f(x) = 2 - x$ on $[0, 2]$ and the point where this average value occurs.
- b) Find $\int_{-\sqrt{5}}^0 \sqrt{5-x^2} \, dx$ (6 marks)
8. Let $f(x) = \frac{2x-9}{x+3}$
- a) Find the intervals where $f(x)$ is increasing and where it is decreasing. Find the local extrema (if any)
- b) Find the intervals where $f(x)$ is concave up and where it is concave down. Find the inflection points (if any).
- c) Find the horizontal and vertical asymptotes (if any)
- d) Sketch the graph (8 marks)